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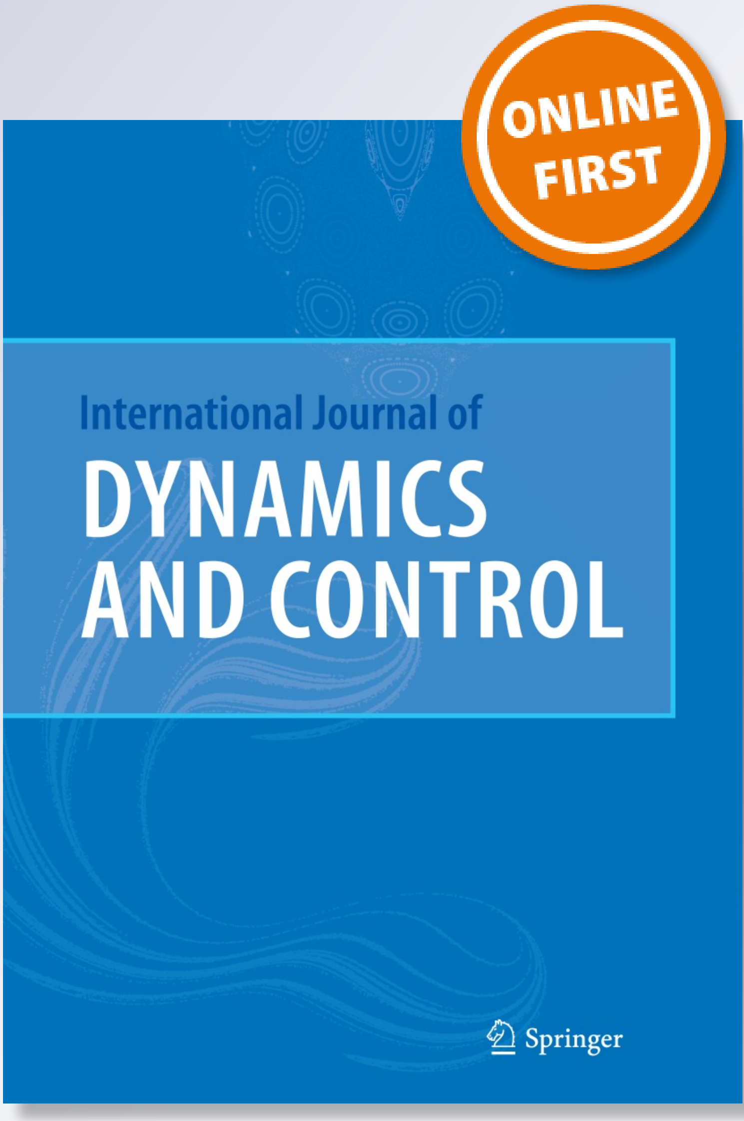
International Journal of Dynamics and Control

ISSN 2195-268X

Int. J. Dynam. Control
DOI 10.1007/s40435-014-0121-6



Springer

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A new state and perturbation observer based sliding mode controller for uncertain systems

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Received: 18 February 2014 / Revised: 13 June 2014 / Accepted: 28 July 2014

Abstract This paper proposes a new continuous sliding mode controller and observer for perturbed systems. The controller, which does not need any knowledge of bounds of any perturbation, is estimated using the uncertainty and disturbance estimation technique and implemented with observer states. The stability of the overall observer-controller combination is proved. The controller-observer combination is validated by implementing the scheme for an actual hardware set-up of serial flexible joint in laboratory.

Keywords Sliding mode control · State and perturbation observer · Observer

1 Introduction

Sliding mode control (SMC) is an excellent method for controlling systems with perturbation [1,2] which has found a variety of applications in the fields of robotics, aerospace vehicles, electrical drives and automotive industry to name just a few. If the uncertainties and disturbances satisfy the matching conditions [3] and if their bounds are known then the SMC can guarantee that the system response will be insensitive to these uncertainties and external disturbances. The control in conventional SMC is discontinuous and requires the full state vector for its implementation. All these points mentioned here, are well known restrictions, concerns or problems in the field of sliding mode control, for which researchers have provided a variety of solutions [4–6].

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Interestingly, newer solutions to all these problems keep coming. When the bounds of uncertainties are not known or are difficult to find, a combination of adaptive control with SMC [7,8] can be used. The problem of matching conditions can be solved for a certain class of systems using the backstepping technique [9]. The bad effects of discontinuous control in causing excessive wear and tear of components and exciting unmodelled dynamics can be overcome by employing a smooth approximation of the discontinuous control as shown in [10,11]. As far as the problem of obtaining the state vector is concerned, an observer that estimates the states in the presence of perturbation is required. Slotine et al. [12] proposed an observer in which a discontinuous feedback from the output error was used in conjunction with a continuous feedback of output error. Jiang and Wu [13] proposed a nonlinear adaptive control through a sliding mode state and perturbation observer. Walcott and Zak [14], Edwards and Spurgeon [1,15] have proposed discontinuous observers for nonlinear systems. In Spurgeon [16] sliding mode observers based on conventional sliding, higher order sliding and observers used for fault detection have been exhaustively surveyed. The discontinuous observer is extended to discrete systems by Yang et al. [17] and used for implementation of the sliding mode controller. Bejarano and Fridman [18] have proposed a second-order sliding mode observer for linear time invariant systems affected by unknown inputs. Several other results based on higher order sliding mode observers have recently appeared in the literature [19]. Design of a sliding mode observer for sensorless PMSM Control was proposed by Lee and Lee [20], while Xu and Rahman [21] proposed a sliding observer for direct torque controlled synchronous motor drives.

Around 1990, Youcef-Toumi and Ito [22] and Youcef-Toumi and Reddy [23] proposed a method called the time delay control (TDC) for the control of uncertain systems. The

Published online: 13 August 2014

method of TDC works on the assumption that the perturbation affecting a system do not change significantly in small interval of time and therefore measurements in the recent past can be used to get an estimate of these perturbation. The method of TDC can be extended to estimate the states of the system as well, as shown by Chang et al. [24] and Kwon and Chung [25] by proposing Time Delay Observers to give combined state and perturbation estimates. Chang and Lee [26] proposed a model reference sliding mode controller for discrete systems with state and perturbation estimation using a TDC-like approach. Bandyopadhyay and Fulwani [27] used a similar approach for the estimation of perturbation and designed an observer using a multi-rate output feedback strategy. The method of TDC requires numerical differentiation and may result in initial oscillations.

Zhong and Rees [28] proposed an uncertainty and disturbance estimator (UDE) to overcome some of the drawbacks of TDC. The method was extended to SMC by Talole and Phadke [29] and to nonlinear time delay systems with application to continuous stirred tank reactor systems by Kuperman and Zhong [30,31]. Like TDC, the method finds many applications where the perturbation is significant. Application of the method to flexible joint robot manipulator is reported by Patel et al. [32] where the effect of joint flexibility is treated as a disturbance and estimated using the method of UDE.

In this paper, we extend the method of UDE to the problem of estimation of states and perturbation, use the estimated perturbation in the design of a sliding mode controller and prove the stability of the observer-controller combination. In general, the sliding mode observer-controllers proposed in the literature need the bounds of perturbation and the control is discontinuous. It is difficult to know the bounds of perturbation precisely leading to a large control activity. The discontinuous control is known to cause wear and tear of components. In the proposed work both these drawbacks are overcome.

The main contributions of our paper are as follows:

(i) the SMC based on UDE is made implementable when the states are not available
(ii) no knowledge of the bounds of perturbation is required
(iii) state and perturbation observer is designed
(iv) unlike the conventional SMC, the control is continuous
(v) in a certain sense, it removes the restriction of matching conditions on perturbation
(vi) the proposed scheme is validated using actual hardware

The paper is organized as follows: The problem is stated in Sect. 2. Model following SMC design based on UDE is presented in Sect. 3. The Sect. 4 describes the state and perturbation observer design. Controller-observer design is given in Sect. 5 and the stability is discussed in Sect. 6. The efficacy of the proposed controller is illustrated by simulation and experimentation in Sect. 7 and the paper is concluded in Sect. 8.

2 Problem formulation

Consider a single-input, single-output system

$$\dot{x}(t) = Ax(t) + Bu(t) + \Delta Ax(t) + \Delta Bu(t) + d(x,t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the control input, $y(t) \in \mathbb{R}$ is the output, $d(x,t) \in \mathbb{R}^n$ is a perturbation vector containing state and time dependent unknown nonlinearities as well as external unmeasurable disturbances. The matrices A, B and C are known matrices of suitable dimensions while ΔA, ΔB represent the parametric uncertainties.

Assumption 1 The uncertainties ΔA, ΔB and the disturbance $d(x,t)$ satisfy the matching conditions, expressed as:

$$\Delta A = BD, \quad \Delta B = BE, \quad d(x,t) = Bv(x,t) \quad (3)$$

where D, E are unknown matrices of suitable dimensions and $v(x,t) \in \mathbb{R}$ is an unknown function.

The Assumption 1, first stated by Drazenovic [3], is a standard assumption in the literature on sliding mode control. Using the Assumption 1, the uncertainties and disturbances can be combined into a perturbation term and defined as $e(x,u,t) \in \mathbb{R}$, and given by

$$e(x,u,t) = Dx(t) + Eu(t) + v(x,t) \quad (4)$$

The system (1) can now be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + Be(x,u,t) \quad (5)$$

Assumption 2 (i) The pair (A,B) is controllable.
(ii) The pair (A,C) are observable.
(iii) The triple (A,C,B) has no invariant zeros, i.e. for all $\lambda \in \mathbb{C}$

$$\text{Rank}\begin{bmatrix} \lambda I - A & -B \\ C & 0 \end{bmatrix} = n+1 \quad (6)$$

Assumption 3 The perturbation $e(x,u,t)$ is continuous and satisfies

$$\left|\frac{d^l e(x,u,t)}{dt^l}\right| \leq \mu \quad \text{for} \quad l = 1,2,\ldots,r \quad (7)$$

where μ is a positive unknown number.

Thus the assumption requires that the derivatives of $e(x, u, t)$ up to some order r be bounded but the bound is not required to be known. The assumption admits a fairly large class of perturbation that can be estimated using the estimation scheme described in Sect. 3.2. In this paper we will consider the problem of designing a SMC for the system (5) when only the output $y(t)$ is available for measurement.

All norms $\|\cdot\|$ considered here are euclidian norms. For example, a vector

$$z = [z_1\ z_2\ \ldots z_n]^T,$$
$$\|z\| = \sqrt{z_1^2 + z_2^2 + \cdots z_n^2}.$$

3 Design of control

In order to increase the readability of the paper, the sliding mode control for the uncertain system (5) is designed by assuming that the plant state vector x is available for measurement. Since, in general, the state x is not available in practice, we will design an observer in the next section and implement the control using the estimated states. The control is designed on the lines of that by Talole and Phadke [29], where the control is implemented using the full state vector whereas in the proposed scheme the control is implemented using the estimate of the state vector.

3.1 Model following control

The design of SMC for system (5) such that the system states should follow a reference model given by

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \tag{8}$$

where $x_m(t) \in \mathbb{R}^n$ is the model state, $u_m(t) \in \mathbb{R}$ is the reference input and A_m, B_m are user selected matrices of suitable dimensions such that (8) gives the desired response. Our aim is to design a control such that the plant states x follow the reference model states x_m in spite of perturbation.

Assumption 4 Now we make assumptions on the structure of the model that can be selected to ensure perfect model following.

$$A_m - A = BM \tag{9}$$
$$B_m = BN \tag{10}$$

where M and N are known matrices of appropriate dimensions.

Now select the sliding surface σ

$$\sigma = B^T x + z, \quad z(0) = -B^T x(0) \tag{11}$$

where B^T denotes the transpose of input matrix B and where the auxiliary variable $z \in \mathbb{R}$ is defined as

$$\dot{z} = -B^T(A_m x + B_m u_m) \tag{12}$$

Let the control be defined as

$$u = u_{eq} + u_n \tag{13}$$

where u_{eq} will be designed to cater to the known part of the system, while u_n will be designed to take care of the perturbation. Differentiating the sliding variable σ in (11) and using (5) and (12) we get

$$\dot{\sigma} = B^T(Ax + Bu + Be(x, u, t) - A_m x - B_m u_m) \tag{14}$$
$$= B^T(-BMx - BNu_m + Bu_{eq} + Bu_n + Be(x, u, t)) \tag{15}$$

Selecting

$$u_{eq} = Mx + Nu_m - (B^T B)^{-1} k\sigma \tag{16}$$

where k is a positive constant chosen by the designer,

$$\dot{\sigma} = B^T Bu_n + B^T Be(x, u, t) - k\sigma \tag{17}$$

If u_n is designed in such a way that it negates the effect of the uncertainty $e(x, u, t)$, then it can be seen from (17) that the sliding mode condition will be satisfied and σ will go to zero asymptotically. This in turn, implies that the controlled plant behaves like

$$\dot{x}(t) = A_m x(t) + B_m u_m(t) \tag{18}$$

which assures the model following. Such a control u_n can be designed if we can get a good estimate of $e(x, u, t)$. In the next section, we describe briefly, how such an estimate can be obtained using the method of uncertainty and disturbance estimation given in [28,29].

3.2 Perturbation estimation

From (17) one can write

$$e(x, u, t) = (B^T B)^{-1}(\dot{\sigma} + k\sigma) - u_n \tag{19}$$

If $e(x, u, t)$ is passed through a broadband filter $G_f(s)$, the output of the filter can be considered to be an estimate of the perturbation $e(x, u, t)$ as shown in [28]. We will define the order of the filter as the order of the denominator polynomial of $G_f(s)$. Denoting the estimate of $e(x, u, t)$ as $\bar{e}(x, u, t)$, we can write

$$\bar{e}(x, u, t) = e(x, u, t)G_f(s) \tag{20}$$

Using (19)

$$\bar{e}(x,u,t) = \left[(B^T B)^{-1}(\dot{\sigma} + k\sigma) - u_n\right] G_f(s) \tag{21}$$

Selecting $u_n = -\bar{e}$ and rearranging the terms we get

$$\bar{e}(x,u,t) = \frac{G_f(s)}{1 - G_f(s)} (B^T B)^{-1}(\dot{\sigma} + k\sigma) \tag{22}$$

With a choice of a first-order filter $G_f(s)$ given by

$$G_f(s) = \frac{1}{\tau s + 1}, \quad \tau > 0 \tag{23}$$

It may be noted that using (23), $\frac{G_f(s)}{1-G_f(s)}$ works out to $\frac{1}{\tau s}$, which simplifies (22) to

$$\bar{e} = \frac{(B^T B)^{-1}}{s\tau} \left(\dot{\sigma} + k\sigma\right) \tag{24}$$

or equivalently treating $\frac{1}{s}$ as an integration operator

$$\bar{e} = \frac{(B^T B)^{-1}}{\tau} \left(\sigma + k \int_0^t \sigma \, dt\right) \tag{25}$$

It is interesting to note that although the derivative of σ appears in the expression of $\bar{e}$, it turns out that no differentiation is required to get the estimate $\bar{e}$ as seen in (25).

Thus finally, the control u is given by (13), (16) and

$$u_n = -\frac{(B^T B)^{-1}}{\tau} \left(\sigma + k \int_0^t \sigma \, dt\right) \tag{26}$$

Also worth noting is that the control forces the plant (1) to follow the model (8), without having to use the model states x_m in the control.

Remark 1 The equations in this section use mixed time and frequency domain variables. The usage, not uncommon in the literature, can be justified by interpreting the equations as interaction of signals acting on hardware described by transfer functions. It is possible to write the equations without the mixing of variables but we feel the formulation given here is intuitively more appealing.

The control u and the estimate $\bar{e}$ obtained in this section require the full state vector x, which is not available according to the statement of the problem. We overcome this problem in the next section, by designing an observer.

4 State and perturbation observer

In this section, the control law developed in the last section make implementable by proposing an observer that can estimate the states of the plant (5) and the perturbation in it. Unlike in [29], the perturbation is estimated using only the output of the plant.

Consider an observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + B\hat{e}\big(\hat{x}, u, t\big) + L\big(y(t) - \hat{y}(t)\big) \tag{27}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{28}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the observer state, $\hat{e}(\hat{x}, u, t) \in \mathbb{R}$ is an estimate of the plant perturbation $e(x,u,t)$, L is the observer gain matrix to be designed and $\hat{y}(t) \in \mathbb{R}$ is the observer output. Defining the state estimation error by $\tilde{x}$ and the error in estimation of perturbation as $\tilde{e}$, we have

$$\tilde{x} = x - \hat{x} \tag{29}$$

$$\tilde{e} = e(x,u,t) - \hat{e}\big(\hat{x}, u, t\big) \tag{30}$$

Now subtracting (27) from (5) and using (2) and (28), we get

$$\dot{\tilde{x}} = (A - LC)\tilde{x} + B\tilde{e} \tag{31}$$

It is clear that we need to design an estimation law for $\hat{e}(\hat{x}, u, t)$ such that the estimation error goes to zero asymptotically or at least remains ultimately bounded.

From (5)

$$e(x,u,t) = B^+(\dot{x} - Ax - Bu) \tag{32}$$

where $B^+ = (B^T B)^{-1} B^T$ is the pseudo-inverse of B. If the states of the plant were available for measurement, the estimate of $e(x,u,t)$ could have been obtained as

$$\bar{e}(x,u,t) = G_f(s) B^+(\dot{x} - Ax - Bu) \tag{33}$$

Since the states of the plant are not available, we propose an estimate of $e(x,u,t)$ by replacing x in (33) by its estimate $\hat{x}$ as

$$\hat{e}(\hat{x}, u, t) = G_f(s) B^+(\dot{\hat{x}} - A\hat{x} - Bu) \tag{34}$$

Using (27) in (34) and in view of (2), (28) and (29), we get

$$\hat{e}\big(\hat{x}, u, t\big) = G_f(s)\hat{e}\big(\hat{x}, u, t\big) + G_f(s) B^+ LC\tilde{x} \tag{35}$$

which after some rearrangement and calculation gives

$$\frac{1 - G_f(s)}{G_f(s)} \hat{e}(\hat{x}, u, t) = B^+ LC\tilde{x} \tag{36}$$

If $G_f(s) = \frac{1}{\tau s+1}$ as in (23) then (36) can be written as

$$\dot{\hat{e}}(x,u,t) = \frac{B^+LC}{\tau}\tilde{x} \tag{37}$$

$$= \frac{B^+L}{\tau}\left(y-\hat{y}\right) \tag{38}$$

Differentiating (30) and using (37),

$$\dot{\tilde{e}}(\hat{x},u,t) = -\frac{B^+LC}{\tau}\tilde{x} + \dot{e}(x,u,t) \tag{39}$$

Now combining (31) and (39),

$$\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\tilde{e}}\end{bmatrix} = \begin{bmatrix}A-LC & B\\ -\frac{B^+LC}{\tau} & 0\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{e}\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\dot{e}(x,u,t) \tag{40}$$

$$= (\bar{A}-\bar{L}\bar{C})\begin{bmatrix}\tilde{x}\\ \tilde{e}\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\dot{e}(x,u,t) \tag{41}$$

where

$$\bar{A} = \begin{bmatrix}A & B\\ 0_{1\times n} & 0\end{bmatrix};\ \bar{L} = \begin{bmatrix}L\\ \frac{B^+L}{\tau}\end{bmatrix};\ \bar{C} = \left[C\ 0\right] \tag{42}$$

Under the Assumptions 2 and 3 the eigenvalues of the matrix $(\bar{A} - \bar{L}\ \bar{C})$ can be placed arbitrarily. If the choice of states is made in such a way that the matrices A, B and C are in phase variable canonical form, then also it can be proved that the eigenvalues of $(\bar{A} - \bar{L}\ \bar{C})$ can be placed arbitrarily by selecting the gain matrix $\bar{L}$.

Remark 2 It is known that if a system is represented in phase variable form, the matching conditions are always satisfied. However, in general, it is not possible to make such a choice because it is not always possible to find sensors to measure all the states. As the order of the system increases, it becomes increasingly difficult to get the required sensors. With the proposed state and perturbation observer, it is possible to choose the states in the phase variable form because sensors are needed to measure only the output. In this sense the method proposed here removes the restriction of matching conditions.

5 Observer-controller combination

The schematic of the proposed observer-controller scheme is shown in Fig. 1. The proposed observer gives the estimates of the plant states x and the perturbation $e(x,u,t)$, with the results that the control developed in Sect. 3.2 can now be made implementable by replacing x by $\hat{x}$ and $\bar{e}$ by $\hat{e}$. For the sake of clarity we state how the control of Sect. 3.2 will be implemented. The sliding variable is

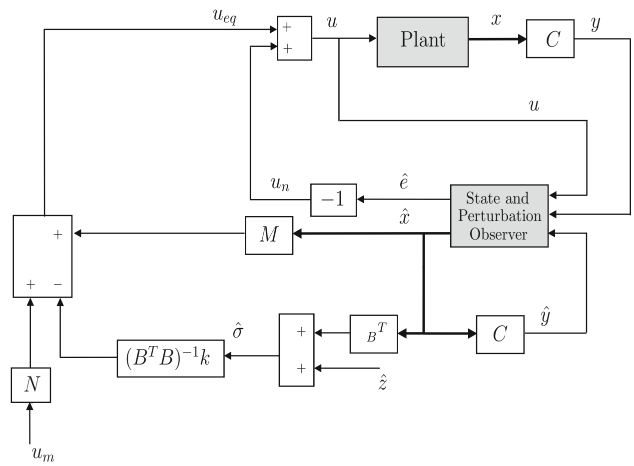

Fig. 1 Schematic of observer-controller combination

$$\hat{\sigma} = B^T\hat{x} + \hat{z},\quad \hat{z}(0) = -B^T\hat{x}(0) \tag{43}$$

$$\dot{\hat{z}} = -B^T\left(A_m\hat{x} + B_m u_m\right) \tag{44}$$

and the components of control u in (13) are implemented as

$$u_{eq} = M\hat{x} + Nu_m - \left(B^TB\right)^{-1}k\hat{\sigma} \tag{45}$$

$$u_n = -\hat{e}\left(\hat{x},u,t\right) \tag{46}$$

where $\hat{e}(\hat{x},u,t)$ is obtained from (37). The dynamics of the uncertain plant (5) under the control given by (13), (45) and (46) works out to

$$\dot{x} = A_m x + B_m u_m + B\tilde{e} - BM\tilde{x} - B(B^TB)^{-1}k\hat{\sigma} \tag{47}$$

Similarly, the dynamics of the sliding variable $\hat{\sigma}$ works out to

$$\dot{\hat{\sigma}} = -k\hat{\sigma} + B^TLC\tilde{x} \tag{48}$$

5.1 Extension to MIMO systems

In this section, the extension of the proposed scheme to multi-input systems, is discussed briefly. For a multi-input system

$$\dot{x}(t) = Ax(t) + Bu(t) + Be(x,u,t) \tag{49}$$

$$y(t) = Cx(t) \tag{50}$$

where the dimension of B matrix is $n \times m$ (m is the number of inputs) and those of the perturbation vector e and the control input u are $m \times 1$. One can design the state and perturbation observer for the multi-input case in a manner similar to (27), (28) with appropriate dimensions of the matrices.

The estimation of perturbation can be written as

$$\hat{e} = G_f(s)B^+\left[B\hat{e} + L(y-\hat{y})\right] \tag{51}$$

where $G_f(s)$ now becomes a diagonal matrix of suitable dimension whose diagonal elements are transfer functions. On similar lines, the observer error dynamics works out to

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{e}} \end{bmatrix} = \begin{bmatrix} A - LC & B \\ -\frac{B^+LC}{\tau} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \tilde{x} \\ \tilde{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \dot{e} \tag{52}$$

where I is the identity matrix of dimension m. The sliding variable is defined as

$$\sigma = B^T \hat{x} + \hat{z}, \quad \hat{z}(0) = -B^T \hat{x}(0) \tag{53}$$

where the sliding variable $\hat{\sigma}$ has dimension $m \times 1$. The reference model should be selected such that the matching conditions (9), (10) are satisfied.

For this case, an additional assumption that $(B^T B)^{-1}$ exists, is needed. One can derive the control input as

$$u_{eq} = M\hat{x} + Nu_m - (B^T B)^{-1} k\sigma \tag{54}$$

$$u_n = -\hat{e} \tag{55}$$

where the linear gain k now becomes diagonal matrix of dimension $m \times m$, $k = \mathrm{diag}(k_1, k_2, \ldots, k_m)$.

Extending this case to multi-output plants is a topic for future work.

6 Stability

Consider the dynamics of the errors in the state and perturbation estimation given by (40). Denoting

$$w = \begin{bmatrix} \tilde{x} \\ \tilde{e} \end{bmatrix} \text{and } A_o = \begin{bmatrix} A - LC & B \\ -\frac{B^+LC}{\tau} & 0 \end{bmatrix} \tag{56}$$

The dynamics (40) can be written as

$$\dot{w} = A_o w + [0_{1\times n} \quad 1]^T \dot{e}(x, u, t) \tag{57}$$

By virtue of the Assumptions 2, it is possible to select the observer gains in such a way that A_o will have eigenvalues at any desired locations in the left half plane. When all the eigenvalues of A_o are negative, one can always find a positive definite matrix P such that,

$$PA_o + A_o^T P = -Q \tag{58}$$

for a given positive definite matrix Q. Let λ_1 be the smallest eigenvalue of Q.

Consider a Lyapunov function

$$V(w, t) = w^T P w \tag{59}$$

Evaluating $\dot{V}$ along (57) and in view of Assumption 3

$$\dot{V} = w^T \left(PA_o + A_o^T P\right) w + 2w^T P [0_{1\times n} \quad 1]^T \dot{e} \tag{60}$$

$$= -w^T Q w + 2w^T P [0_{1\times n} \quad 1]^T \dot{e} \tag{61}$$

$$\le -\lambda_1 \|w\|^2 + 2\|w\| \|P\| \mu \tag{62}$$

Therefore, $\|w\|$ will ultimately remain in a ball of radius $\frac{2\|P\|\mu}{\lambda_1}$, which guarantees that

$$\|\tilde{x}\| \le \frac{2\|P\|\mu}{\lambda_1} \quad \text{and} \quad |\tilde{e}| \le \frac{2\|P\|\mu}{\lambda_1} \tag{63}$$

In view of (48), (47) and (63), it is easy to prove that $\hat{\sigma}$ and the model following error will also be ultimately bounded. All the bounds go to 0 when $\mu = 0$, in which case the asymptotic stability of $\tilde{x}$, $\tilde{e}$, $\hat{\sigma}$ and the model following error is assured.

The observer developed in Sect. 4 can be extended further. For example, when $r = 2$ in Assumption 3, it is possible to estimate $e(x, u, t)$ and $\dot{e}(x, u, t)$ by choosing a second order filter $G_f(s)$ as

$$G_f(s) = \frac{\tau_2 s + 1}{\tau_1 s^2 + \tau_2 s + 1} \tag{64}$$

where τ_1 and τ_2 are small positive constants. Working on lines similar to Sect. 4, one can find the observer equations for the estimation of uncertainty

$$\dot{\hat{e}}_1 = \frac{\tau_2}{\tau_1} B^+ LC\tilde{x} + \hat{e}_2 \tag{65}$$

$$\dot{\hat{e}}_2 = \frac{1}{\tau_1} B^+ LC\tilde{x} \tag{66}$$

where $\hat{e}_1$ is the estimate of $e(x, u, t)$. The filter $G_f(s)$ given in (64) and the estimation laws for the lumped uncertainty $e(x, u, t)$ and its derivatives given in (65), (66) can be generalized without much difficulty. It can be proved that under the Assumption 2 the stability of the observer-controller combination for the general case is assured.

6.1 Generalization of the filter

The filter $G_f(s)$ can be generalized to an $r-$ th order filter can be given by,

$$G_f(s) = \frac{\tau_2 s^{r-1} + \tau_3 s^{r-2} + \cdots + \tau_r s + 1}{\tau_1 s^r + \tau_2 s^{r-1} + \tau_3 s^{r-2} + \cdots + \tau_r s + 1} \tag{67}$$

where the coefficients τ_i, $i = 1, 2, \ldots, r$ are selected such that the poles of the transfer function of the filter lie in LHP. The equations of estimations of perturbation and its derivatives can be written as

$$\dot{\hat{e}}_1 = \frac{\tau_r}{\tau_1} B^+ LC\tilde{x} + \hat{e}_2$$
$$\dot{\hat{e}}_2 = \frac{\tau_{r-1}}{\tau_1} B^+ LC\tilde{x} + \hat{e}_3$$
$$\vdots$$
$$\dot{\hat{e}}_{r-1} = \frac{\tau_2}{\tau_1} B^+ LC\tilde{x} + \hat{e}_r$$
$$\dot{\hat{e}}_r = \frac{1}{\tau_1} B^+ LC\tilde{x} \tag{68}$$

where $\hat{e}_1$ is the estimate of perturbation e and $\hat{e}_i, i = 2, 3, \ldots, r$ are the estimates of derivatives of perturbation $e^{(j)}, j = 1, 2, \ldots, r-1$. Defining the perturbation and its derivatives as $e_1 = e,\ e_2 = \dot{e}, \ldots, e_r = e^{(r-1)}$ and defining the estimation errors as

$$\tilde{e}_i = e_i - \hat{e}_i \quad \text{for } i = 1, 2, \ldots, r \tag{69}$$

The dynamics of the perturbation error can be written as,

$$\dot{\tilde{e}}_1 = -\frac{\tau_r}{\tau_1} B^+ LC\tilde{x} + \tilde{e}_2$$
$$\dot{\tilde{e}}_2 = -\frac{\tau_{r-1}}{\tau_1} B^+ LC\tilde{x} + \tilde{e}_3$$
$$\vdots$$
$$\dot{\tilde{e}}_{r-1} = -\frac{\tau_2}{\tau_1} B^+ LC\tilde{x} + \tilde{e}_r$$
$$\dot{\tilde{e}}_r = -\frac{1}{\tau_1} B^+ LC\tilde{x} + e^{(r)} \tag{70}$$

The dynamics of state and perturbation observer can be written in a compact form as

$$\dot{w} = \bar{A}w + Ee^{(r)} \tag{71}$$

where,

$$\bar{A} = \begin{bmatrix} A - LC & B & \cdots & 0 \\ -\frac{\tau_r}{\tau_1} B^+ LC & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ -\frac{1}{\tau_1} B^+ LC & 0 & \cdots & 0 \end{bmatrix};\ E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix};$$

$$w = \begin{bmatrix} x \\ \tilde{e}_1 \\ \vdots \\ \tilde{e}_r \end{bmatrix}$$

Fig. 2 Quanser serial flexible joint set-up

7 Illustrative example

The effectiveness of the proposed observer controller scheme is evaluated by applying it to a practical system consisting of a serial flexible joint [33]. The objective of the proposed scheme is to track a reference model in spite of perturbation.

7.1 Experimental setup

The serial flexible joint shown in Fig. 2, consists of a DC motor unit with harmonic drive through which the link is connected and fitted with an optical encoder for measuring angular position of the flexible link. The unit used in this experiment does not have a sensor to measure the link velocity, acceleration and jerk.

7.2 Flexible joint manipulator

The equations of motion are taken as [34]

$$I\ddot{q}_1 + MgL\sin(q_1) + K(q_1 - q_2) = 0$$
$$J\ddot{q}_2 - K(q_1 - q_2) = u \tag{72}$$

where I is the equivalent link inertia, J is the equivalent motor inertia, K is the effective spring stiffness constant of the elastic joint, u is the input toque, M is the mass of flexible link and L is the effective length of the link, q_1 and q_2 are the link and motor shaft displacements respectively.

In state space form, the serial flexible joint can be represented as,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{K_{s_1}}{J_{11}} x_3 - \frac{K_{s_1}}{J_{11}} x_1 - \frac{B_{11}}{J_{11}} x_2 - \sin x_1$$
$$\dot{x}_3 = x_4$$

Table 1 Nominal parameter of flexible joint manipulator [33]

Parameters	Description	Nominal value	Unit
K_{t_1}	Drive torque constant	0.119	N m/A
K_{s_1}	Torsional stiffness constant	9	N m/rad
J_{11}	Actuated transition equivalent		
	Moment of inertia	931E−6	kg m^2
J_{12}	Load transition equivalent		
	Moment of inertia	0.230	kg m^2
B_{11}	Actuated transition equivalent		
	Viscous damping coefficient	4.5	N m s/rad
B_{12}	Load transition equivalent		
	Viscous damping coefficient	0.07	N m s/rad

$$\dot{x}_4 = \frac{K_{s_1}}{J_{12}}x_1 - \frac{K_{s_1}}{J_{12}}x_3 - \frac{B_{12}}{J_{11}}x_4 + \frac{K_{t_1}}{J_{11}}u \tag{73}$$

where system internal states $x = [x_1, x_2, x_3, x_4]^T$ are the link position, the link velocity, the motor shaft position, the motor shaft velocity respectively, u is the control input to the motor. Table 1 shows the nominal parameter values considered for simulation and experimental validation. For flexible joint simulation, 10 % of uncertainties in nominal system parameters are considered.

Converting plant dynamics into phase variable canonical form and defining plant's new states as $z_1 = y$, $z_2 = \dot{y}$, $z_3 = \ddot{y}$, $z_4 = \dddot{y}$, the dynamic equations can be represented as

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= z_4 \\
\dot{z}_4 &= -\frac{k_{s_1}}{J_{11}}z_2 - \left(\frac{k_{s_1}(J_{11}+J_{12})}{J_{11}J_{12}}\right)z_3 \\
&\quad - \left(\frac{B_{12}}{J_{11}} + \frac{B_{11}}{J_{11}}\right)z_4 + \frac{K_{t_1}}{J_{11}}u
\end{aligned} \tag{74}$$

7.3 Simulation results

In this section, two cases are considered to show the efficacy of the proposed scheme. In the first case, the simulation results obtained with the proposed system are compared with those obtained with the Luenberger observer and in the second case, the improvement in the estimate of perturbation obtained by implementing a second order filter is shown.

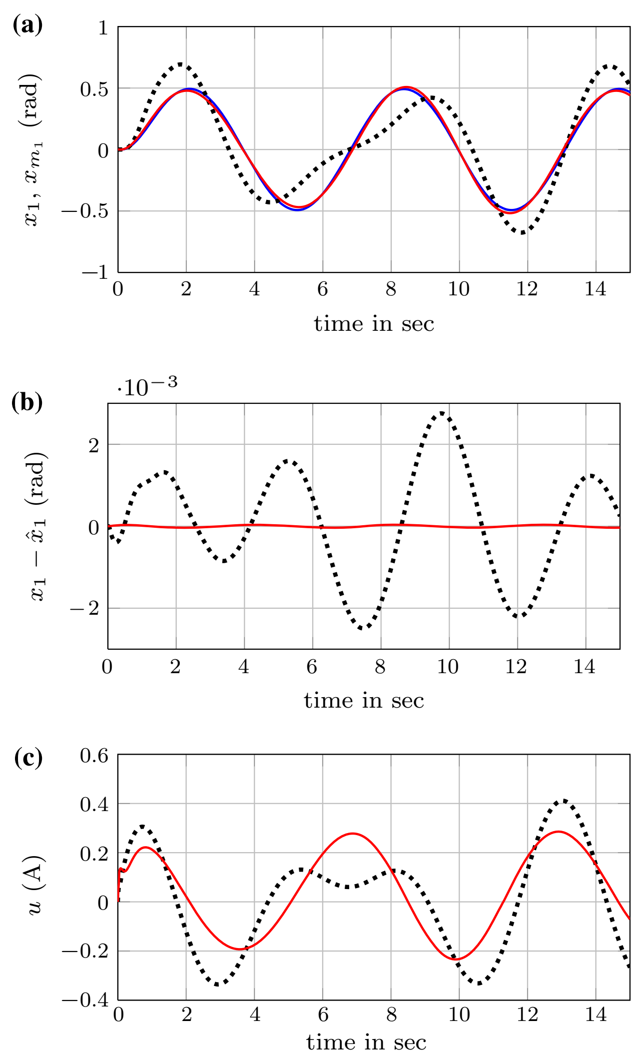

Fig. 3 Simulation results (Case-I): comparative performance of the Luenberger observer (*dotted line*) and the proposed observer with first order filter (in *red*) in the presence of 10 % uncertainty **a** x_1 and x_{m_1} (in *blue*), **b** $x_1 - \hat{x}_1$, **c** u. (Color figure online)

7.3.1 Case-I

In this case, the initial conditions of reference model $x_m(0)$ and observer are $\hat{x}(0)$ set to $\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^T$. In this case, 10 % parametric uncertainties over the nominal matrices A and B are introduced.

Figure 3a shows comparative plot of plant state x_1 and reference model x_{m_1} state. It can be seen that with the proposed scheme, the plant state x_1 tracks the reference states x_{m_1} very closely, whereas the corresponding results with the Luenberger observer are poor. The Fig. 3b shows the observer estimation error, where it can be seen that proposed scheme has much better accuracy compared to the Luenberger observer

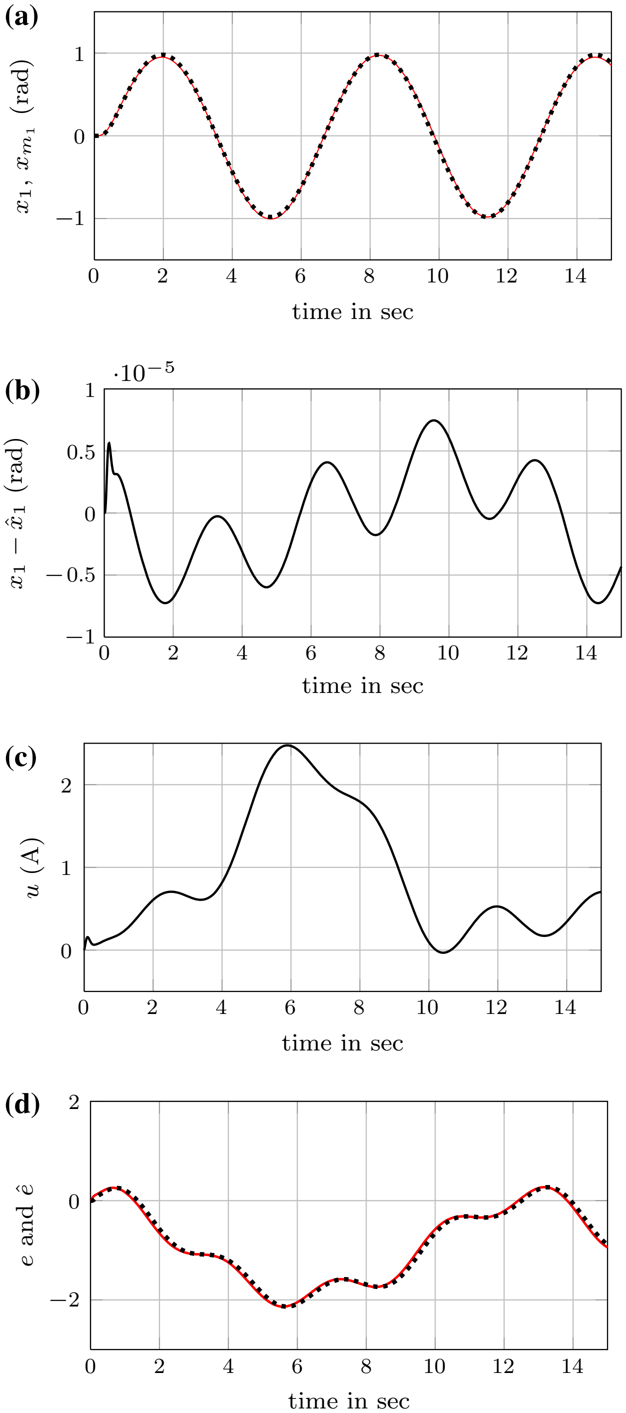

Fig. 4 Simulation results: performance of the proposed observer with first order filter in the presence of 10 % uncertainty and complex disturbance **a** x_1 (*solid line*) and x_{m_1} (*dotted line*), **b** $x_1 - \hat{x}_1$, **c** u, **d** e (*solid line*) and $\hat{e}$ (*dotted line*)

design. The comparative plot of the control input is shown in Fig. 3c. Similar results were obtained with other types of disturbances but are not shown here to save space.

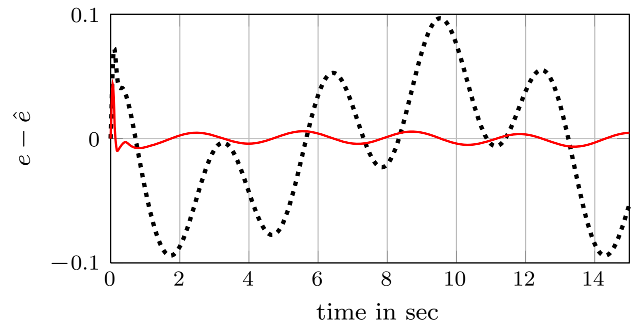

Fig. 5 Simulation result (Case-II): perturbation estimation error using first order filter (*dotted line*) and second order filter (*solid line*)

7.3.2 Case-II

In this case, the initial conditions of reference model $x_m(0)$ and observer $\hat{x}(0)$, perturbation in A and B are considered same as the previous case and additional disturbance $0.25\sin(2t) + \cos(0.5t) - 1$ is introduced in the input channel of the plant.

Figure 4a shows comparative plot of plant state x_1 and reference model x_{m_1} state. It can be seen that with the proposed observer, the plant state x_1 tracks the reference states x_{m_1} very closely. The Fig. 4b and d shows the observer estimation error in output, perturbation and its estimation respectively. The plot of the control input is shown in Fig. 4c. The comparison of perturbation estimation error using first and second order filter is shown in Fig. 5. Similar results were obtained with different level of perturbations but are not shown here to save space.

7.4 Experimental results

Next the proposed strategy was implemented on the practical setup shown in Fig. 2. The plots of the plant state x_1, observer state $\hat{x}_1$ and model state x_{m1} in Fig. 6a show a good match between the plant, observer and model position. Fig. 6b shows the estimation error of plant output x_1 and observer output $\hat{x}_1$. The control input and estimation of the perturbation is plotted in Fig. 6c and d respectively.

In order to improve the accuracy of estimation further, the observer was implemented with second order filter, the results of which are shown in Fig. 7. The errors in model following $x_1 - x_{m1}$, with the first and the second order filter are shown in Fig. 8a, which shows a marked improvement obtained by the second order filter. The error in model following drops by a factor of 3 when a second order filter was employed. It is worth noting from Fig. 8b that the improvement in model following is brought about without an appreciable change in the magnitude of control.

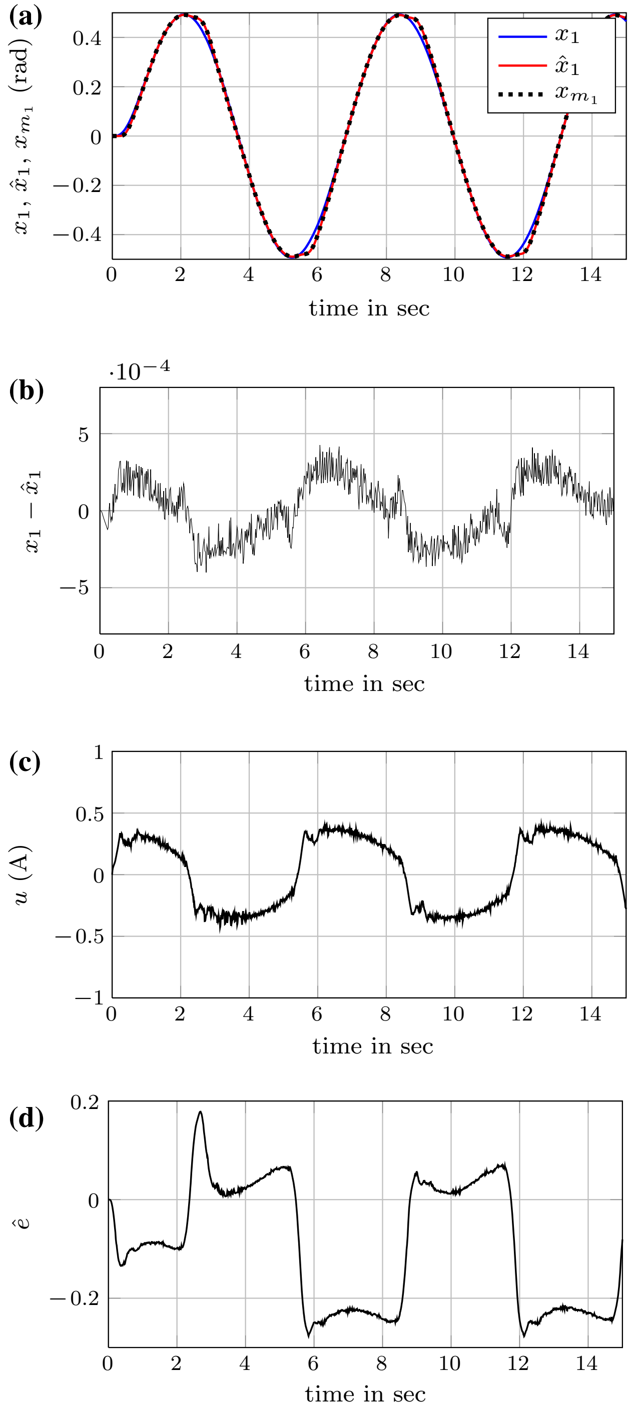

Fig. 6 Performance using first order filter in the presence of parametric uncertainty and sinusoidal disturbance **a** plot of first state of plant, observer (estimation of link position) and model, **b** plot of observer estimation error in output, **c** plot of control input, **d** plot of estimates of perturbation

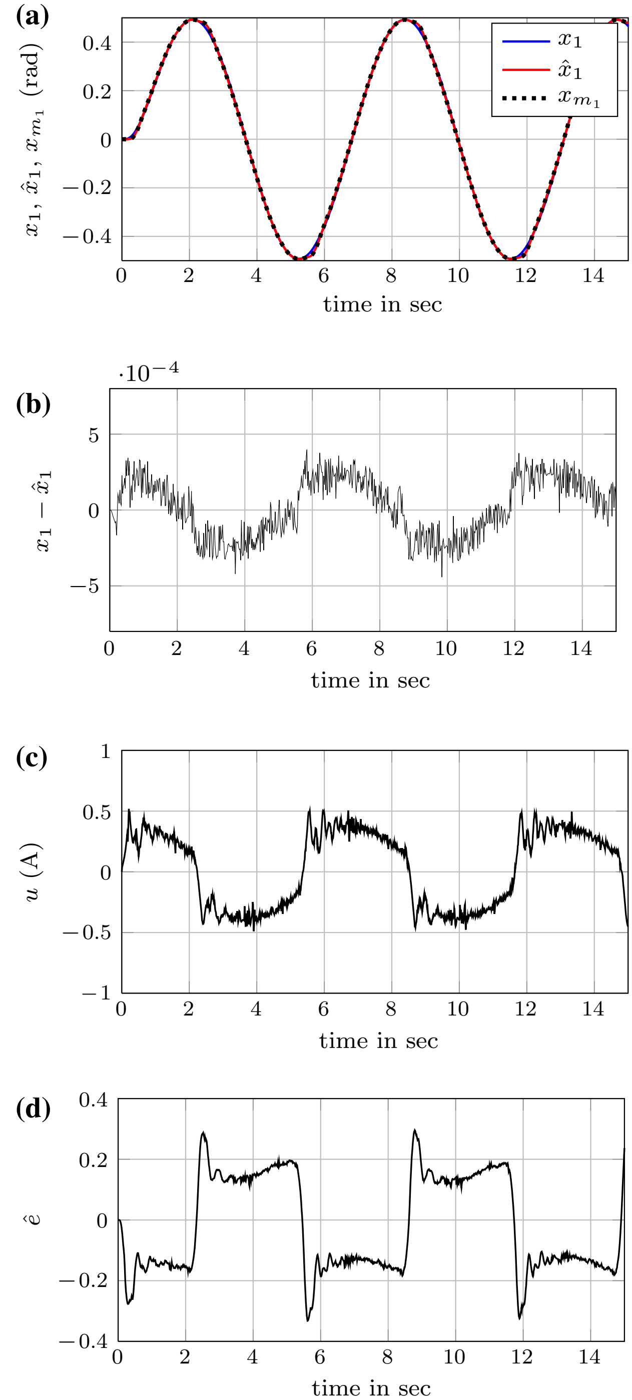

Fig. 7 Performance using second order filter in the presence of parametric uncertainty and sinusoidal disturbance **a** plot of first state of plant, observer (estimation of link position) and model, **b** plot of observer estimation error in output, **c** plot of control input, **d** plot of estimates of perturbation

Apart from the presented simulation and experimental results, the proposed scheme was also validated in the presences of different perturbation with similar performance. To save space, the results for these cases are not shown.

8 Discussion and conclusion

In comparison to sliding mode observer-controllers reported in the literature, the proposed scheme has many advantages.

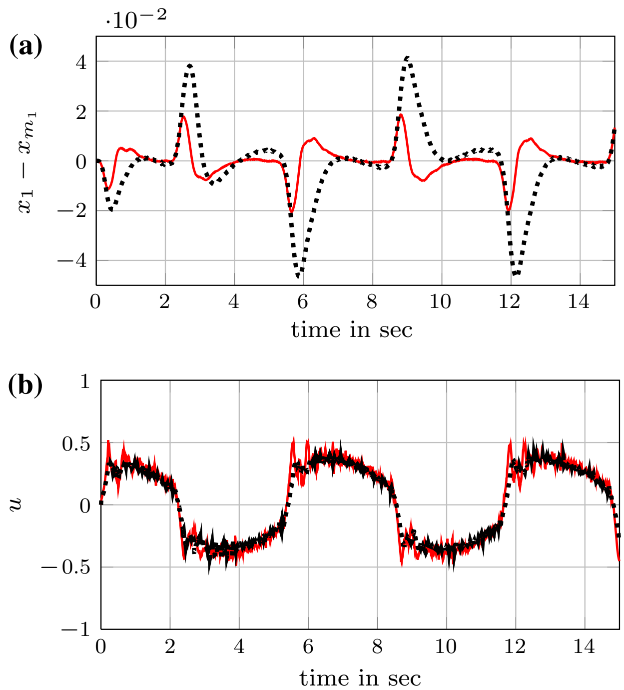

Fig. 8 Comparative plot of error in model following and control input for proposed observer with first order filter (*dahsed line*) and second order filter (*solid line*)

The control in the proposed scheme depends on the estimate of the instantaneous value of the perturbation and is therefore smaller than those in the schemes which depend on the bound of perturbation. Further the control in the proposed scheme is smooth. The accuracy of estimation of states and perturbation can be improved by using a filter of higher order. The proposed scheme is well suited to a variety of practical applications like manipulators, position control systems, automotive and aerospace control. The estimation error in the proposed scheme is proportional to rate of change of perturbation. Therefore for a systems in which the perturbation is fast varying, one needs a higher order filter which increases the complexity of the scheme. The proposed scheme is validated by simulation and experimentation on an actual hardware. It is shown that the estimates of perturbation can be improved further by using a second order filter which in turn improves the overall performance of the observer-controller combination. Using only the position of the link, both the perturbation the link velocity, motor shaft position, motor shaft velocity as well as the perturbation is estimated. The ultimate boundedness of all signals is demonstrated. The comparison of the proposed scheme with the Luenberger observer-controller scheme shows the ability of the proposed scheme to cope up with the perturbation.

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